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Numerical simulation of blood modeled as a fluid-particulate mixture

Krishna Chandran[a], Indranil Saha Dalal[b*], Kazuya Tatsumi[c] and Krishnamurthy Muralidhar[a]
[a]Department of Mechanical Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India
[b]Department of Chemical Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India
[c]Department of Mechanical Engineering and Science, Kyoto University, Kyoto 606-8501, Japan
* Corresponding author, E-mail: indrasd@iitk.ac.in

Abstract

Migration of red blood cells (RBC) towards the center of the capillaries from the walls due to shear-induced diffusion (SID) is a recently studied phenomenon. It results in a cell-free layer near the walls of the capillaries. While numerous works are available in literature for the SID of particulate laden flow, a blood rheology model which predicts accurately the RBC concentration distribution along with a mass conserving velocity field has not been addressed. In the present study, a mathematical model for the transport of red blood cells (RBC) represented as a particulate is presented. It proposes a coupled species-momentum transport model to predict the RBC distribution inside a tubular geometry and in patient-specific human arteries.

Keywords : Shear-induced diffusion, red blood cells, particulate, hematocrit, cell free layer

1. Introduction

Blood is a complex fluid primarily constituting of erythrocytes (RBC), leukocytes (WBC) and platelets. Blood flow inside arteries and capillaries can cause WBCs and platelets to separate from RBCs and tend to move towards the wall by a process called margination. On the other hand, RBCs tend to migrate towards the center of the arteries which cause RBC depletion near the wall leading to what is known as the cell free layer at the wall [1]. Under normal physiological conditions blood behaves as a suspension and hence the interaction between the blood particulates especially RBCs become more important to understand its rheology. RBC being a non-spherical and deformable particle, it tends to have an irreversible effect due to mutual interactions resulting in anisotropic non-linear shear induced diffusion (SID) [2]. Only a few experimental measurements [3] of SID coefficient are available in the literature since the diffusivity at that scale is small. Rusconi and Stone [4] conducted experiments to show strong cross-stream shear induced diffusion for platelet-like particles. Their experiments provided quantitative results for a wide range of shear rates. Grandchamp et al. [5] developed a mathematical model for the transport of RBCs from experiments. Their results showed that the diffusion of RBCs are primarily in the shear and vorticity directions.

Understanding rheology is important in deriving appropriate constitutive models for blood. Apostolidis and Berris [6] proposed a hematocrit dependent stress-strain constitutive model to characterize the non-Newtonian behaviour of blood. The study showed that the Casson constitutive model fits the best where the apparent viscosity is dependent on the hematocrit concentration. In the present work, a coupled RBC-momentum transport model for blood is proposed where the velocity and pressure fields are obtained from the Navier-Stokes equations and the RBC concentration is obtained from the model developed by Grandchamp et al. [5]. The momentum equation is coupled to the RBC transport equation through a hematocrit dependent fluid viscosity model [6].

2. Mathematical model

Following [5], the governing equations for velocity, pressure and RBC concentration are given as:

$$\left(\frac{\partial(\rho\mathbf{u})}{\partial t}+\nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u})\right)=-\nabla\cdot(p\mathbf{I})+\nabla\cdot(\mu(\phi)\nabla\mathbf{u}) \tag{1}$$

$$\nabla\cdot(\rho\mathbf{u})=0 \tag{2}$$

$$\frac{\partial\phi}{\partial t}+u(y,z)\frac{\partial\phi}{\partial x}=-\frac{\partial}{\partial y}J_y-\frac{\partial}{\partial z}J_z \tag{3}$$

Equations (1) and (2) are the momentum and the continuity equations whereas Eq. (3) gives the RBC transport equation where the fluxes J_y and J_z are given as [5]:

$$J_y=-R^2\Gamma^{-1}\phi\left\{\left(f_3u_z^2+f_2u_y^2\right)\frac{\partial\phi}{\partial y}+\left(f_2-f_3\right)u_yu_z\frac{\partial\phi}{\partial z}\right\};\ J_z=-R^2\Gamma^{-1}\phi\left\{\left(f_3u_y^2+f_2u_z^2\right)\frac{\partial\phi}{\partial z}+\left(f_2-f_3\right)u_yu_z\frac{\partial\phi}{\partial y}\right\}$$

The constants f_2 and f_3 are dimensionless diffusivities in the shear and vorticity directions. Further, R is the radius of RBC and Γ is the magnitude of the strain rate tensor [5]. The hematocrit-dependent viscosity model is given as:

$$\mu_{eff} = \left[\sqrt{\frac{\tau_y}{\dot{\gamma}}} + \sqrt{\mu_C} \right]^2 ; \ \mu_C = n_p \left(1 + 2.0703 \times \phi + 3.722 \times \phi^2\right) \times \exp\left(-7.0276 \left(1 - \frac{T_0}{T} \right) \right) \quad (4)$$

The constants T_0=296.16 K and n_p=1.67×10^{-3} Pa-s.

3. Results and Discussion

Figure 1 shows the outcome of validation studies carried out in the present work. As a first step, the RBC concentration profile given in [7] for the same geometry was extracted and was curve fitted to obtain a close match as shown in Fig. 1a. Next, this RBC concentration distribution was given as the initial condition and Eqs. (1) and (2) were solved simultaneously with the RBC dependent viscosity model Eq. (4). Figure 1b shows that the computed velocity profile is in good agreement with the result obtained in [7] for blood flow inside a pipe of diameter 40 μm. The mass conserving velocity field is shown to match well with [7].

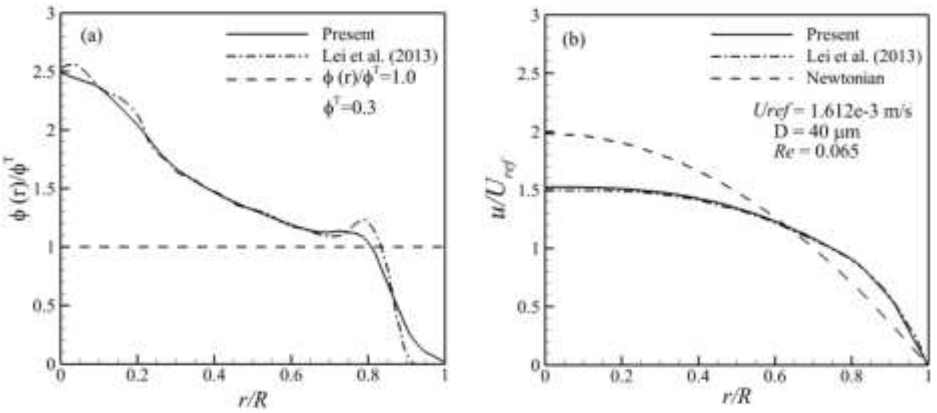

Figure 1: Comparison of (a) RBC concentration profile and (b) velocity profile for a flow inside a tube of diameter 40 μm with the literature [7].

4. Conclusions

A mathematical model is proposed to solve a coupled nonlinear species-momentum transport model. Velocity and pressure fields are solved using the Navier-Stokes equations while the RBC transport equation is solved using the mathematical model proposed by Grandchamp et al. [5]. Casson hematocrit dependent stress-strain constitutive model is used to couple the momentum and RBC transport equations via the effective viscosity. For a prescribed hematocrit distribution, the velocity profile is shown to match well with the literature for a flow through a circular tube. Further simulations will consider pulsatile flow and non-circular geometries.

Acknowledgements

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References

[1] A. Kumar, M.D. Graham, Mechanism of margination in confined flows of blood and other multicomponent suspensions. *Physical Review Letters*, *109*(10), p. 108102, 2012.

[2] F.R. Da Cunha, E.J. Hinch, Shear-induced dispersion in a dilute suspension of rough spheres. *Journal of Fluid Mechanics*, *309*, pp. 211-223, 1996.

[3] I.E. Zarraga, D.T. Leighton Jr., Measurement of an unexpectedly large shear-induced self-diffusivity in a dilute suspension of spheres. *Physics of Fluids*, *14*(7), pp. 2194-2201, 2002

[4] R. Rusconi, H.A. Stone, Shear-induced diffusion of plate-like particles in microchannels. *Physical review letters*, *101*(25), p.254502, 2008

[5] X. Grandchamp, G. Coupier, A. Srivastav, C. Minetti, T. Podgorski, Lift and down-gradient shear-induced diffusion in red blood cell suspensions. *Physical Review Letters*, *110*(10), p. 108101, 2013.

[6] A.J. Apostolidis, A.N. Beris, Modeling of the blood rheology in steady-state shear flows. *Journal of Rheology*, *58*(3), pp.607-633, 2014

[7] H. Lei, D.A. Fedosov, B. Caswell, G.E. Karniadakis, Blood flow in small tubes: quantifying the transition to the non-continuum regime. *Journal of Fluid Mechanics*, *722*, pp. 214-239, 2013.